Numerical Study on Solutions for Hyperbolic $p$-System with Nonlinear Damping

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Abstract

In this talk, we study numerically on the solution behavior for the initial-boundary value problem to the $2\times2$ hyperbolic $p$-system with nonlinear damping,

$$\begin{cases}
v_t - u_x = 0, \\
u_t + p(v)_x = -\alpha u - \beta |u|^{q-1} u, \\
(v, u)|_{t=0} = (v_0, u_0)(x) \to (v_+, u_+) \text{ as } x \to +\infty, \\
u|_{x=0} = 0, \quad u_+ \neq 0,
\end{cases} \quad (x, t) \in \mathbb{R}_+ \times \mathbb{R}_+,$$

where, $p(v) > 0$ and $p'(v) < 0$, $\alpha > 0$, $q \geq 2$ and $v_+ > 0$. When $\beta > 0$, or $\beta < 0$ but $|\beta| < \frac{\alpha}{|u_+|^{q-1}-1}$, the solution $(v, u)(x, t)$ was proved to globally exist and to converge to its corresponding diffusion wave

$$\begin{cases}
\bar{v}_t - \bar{u}_x = 0, \\
p'(\bar{v})_x = -\alpha \bar{u}, \\
\bar{v}|_{t=0} = \bar{v}_0(x) \to v_+ \text{ as } x \to +\infty, \\
\bar{u}|_{x=0} = 0,
\end{cases} \quad (x, t) \in \mathbb{R}_+ \times \mathbb{R}_+,$$

with a specially selected initial data $\bar{v}_0(x)$.

In this work, we use central-upwind schemes to simulate numerically the solution convergence to diffusion waves. Indeed, if the initial perturbation is in the weighted space $L^{1+\gamma}(\mathbb{R}_+)$ with the best choice of $\gamma = \frac{1}{4}$, our numerical results confirm that $\|v - \bar{v})(t)\|_{L^\infty(\mathbb{R}_+)} = O(1)t^{-\frac{9}{8}}$ as stated in [Mei] and show that $\|(u-\bar{u})(t)\|_{L^\infty(\mathbb{R}_+)} = O(1)t^{-13/8}$ which order is better than the theoretical result in [Mei]. This numerical result therefore indicates that further theoretical investigation on
convergence rate is needed.

In addition, when $\beta < 0$ and $|\beta| > \frac{\alpha}{|u_+|/\sigma - 1}$, our numerical simulations also demonstrate that the solution $u$ will blow up at a finite time and confirm the conjecture in [Mei] that the solution $v$ doesn’t blow up.

Keywords: hyperbolic $p$-system, central-upwind schemes, nonlinear damping, diffusion waves, convergence rates, blowup.

[Mei] Ming Mei, Diffusion Waves for Hyperbolic $p$-System with Nonlinear Damping, talk at Providence University, December 31, 2008