Spectral continuation methods using Chebyshev polynomials for nonlinear Schrödinger equations

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Abstract

We study efficient spectral-Galerkin continuation methods (SCM) and pseudo-spectral continuation methods (PSCM) for the numerical solutions of nonlinear eigenvalue problems (NEP). We show that the SCM and PSCM are equivalent for solving NEP if we neglect the integration errors in the quadrature formula. The manifest advantage of the SCM and PSCM over the finite difference methods (FDM) or finite element methods (FEM) for solving the nonlinear Schrödinger equations (NLS) is that in the former the target points on the primary solutions of the NLS are close to the eigenvalues of the associated linear eigenvalue problems. Thus only a few continuation steps are required to reach the target points. This advantage makes both the SCM and PSCM very robust and competitive compared to the recently developed accelerated imaginary-time evolution methods (AITEMs). Extensive numerical experiments were executed to verify the effectiveness and robustness of the proposed numerical methods.

Keywords: (pseudo-)spectral methods; Chebyshev polynomials; Gross-Pitaevskii equation; Bose-Einstein condensation; periodic potential.

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